Dynamic modelling and parameter identification of a three-degree-of-freedom spherical actuator

Xingming Wu¹, Fanghong Guo¹, Jingmeng Liu¹, Weihai Chen¹ and Changyun Wen²

Abstract
This paper addresses both dynamic modelling and dynamic parameter identification of a permanent magnet spherical actuator, which is capable of performing three-degree-of-freedom (DOF) motion in one single joint. The dynamic model of the spherical actuator is derived from Lagrange’s equations, but the parameters, called dynamic parameters, in the model are usually uncertain. Then the dynamic model is represented in a form that is linear in these parameters. A new identification method based on the output error (OE) method and recursive least square (LS) estimation is proposed to identify the parameters. This method only requires the current measurement of the stator coils in the identification procedure, which greatly simplifies the experimental process and improves the identification accuracy. Lastly, simulation and experimental results illustrate the effectiveness of the proposed method and its robustness to external disturbances. The proposed method can be also applied to other electromagnetic driving spherical actuators.

Keywords
Dynamic modelling, inverse dynamics, parameter identification, spherical actuator

Introduction
Recently, research on spherical actuators has been very active in the area of robotics for their compact structure and ability of three-degree-of-freedom (3-DOF) motion with a fast dynamic response. Conventionally, multi-DOF motion is accomplished by combining several single-DOF motors in series or parallel. This has a number of drawbacks, such as having singularities and backlashes within workspaces. The idea of employing multi-DOF actuators overcomes these drawbacks. The first multi-DOF actuator was built by Williams and Laithwaite in 1950s (Williams et al., 1959), with a capability of realizing 2-DOF motion in one joint. Since then, many different kinds of spherical actuators have been proposed, such as a cable-driving spherical actuator by Nagasawa and Honda (2000), spherical ultrasonic actuator by Takemura et al. (2004) and permanent magnet spherical actuator by Wang et al. (2003). Among them, the electromagnetic force driving spherical actuator has received the most attention from researchers. Lee et al. proposed and developed a variable-reluctance spherical actuator (Lee et al., 1996) and spherical wheel motor (Lee et al., 2005), which has a compact size and can realize speed control of the spinning motion. Yan et al. (2006, 2008) designed a permanent magnetic spherical actuator and analytically established a magnetic field model and a torque model for it. A Halbach array permanent magnet spherical motor was proposed by Xia et al. (2009) and it was shown to be more effective in improving air gap field distribution, thus increasing the torque output.

As a spherical actuator is a non-linear and strongly coupled system (Xia et al., 2010), it is rather challenging and complex to propose a control strategy for it. In order to improve the motion accuracy, many researchers have developed a model-based control strategy (Lee et al., 2004; Li, 2009; Wang et al., 1997). However, the design, analysis and the implementation of a model-based controller require accurate knowledge of the spherical actuator’s dynamic model. In Xia et al. (2010) and Son and Lee (2010), a simple dynamic model is derived by assuming that the inertial moment of the two Euler angle axis (α, β) are the same. The dynamic parameters appearing in the equations are approximated via computer-aided design software CAD estimates. However, it is difficult and even impossible to evaluate the accurate dynamic parameters of the spherical actuator. In particular, the friction torque parameters with non-linear dynamic behaviour as well as the gravitational torque cannot be evaluated. In order to obtain more accurate values of the dynamic parameters, an identification technique based on experimental

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data is used in this paper, where the dynamic parameters of the spherical actuator can be obtained from the measurement collected during the motion.

Although there are several theoretical approaches on dynamic model identification of robot manipulators and conventional motors (Wu et al., 2010), the research on multi-DOF actuators is still very limited. In this paper, we will propose an effective and reliable identification algorithm to estimate the dynamic parameters of the spherical actuator, based on the output error (OE) and recursive least square (LS) estimation methods. The proposed identification method is better than other LS estimation methods for its easy operation by only sampling the current of the stator coils during the rotor’s motion. More importantly, identification accuracy can be also improved because less measurement noise is introduced during the experiment. In addition, a dynamic model is derived and studied in detail, and the effects of gravitational torque and the friction torque are considered and re-modelled in the dynamic model.

The rest of the paper is organized as follows. Next, the mechanical structure and the orientation measurement system of the spherical actuator are introduced. Then the dynamic model with friction torque is studied from Lagrange’s equations. A dynamic parameter identification method is proposed, and a control law is discussed at the same time. At last we introduce an identification experiment platform, on which experimental studies are carried out. Both the simulation and the experimental results illustrate the effectiveness of the proposed method.

Spherical actuator

Description of mechanical structure

The spherical actuator shown in Figure 1(a) consists of a ball-shaped rotor with a circular permanent magnet (PM), which provides high flux density for the actuator, and a spherical-shell-like stator with two layers of circumferential air-core coils.

The parameters of its rotor and stator are presented in Tables 1 and 2, respectively. This spherical actuator has the ability to spin continuously about its output shaft in 360° freely, and the rotor can also incline about its equatorial plane to an extreme position that the axes of a PM pole and a coil are aligned. The maximum tilting angle is ±15° for our research prototype. When the coils are activated in order, the spherical actuator can achieve 3-DOF motion in one rotor.

Orientation measurement system

In this research, a spherical joint is designed for the orientation measurement of the rotor. As shown in Figures 2(a) and 2(b), the rotor is connected to the base through the spherical joint. When the rotor generates a motion under the electromagnetic force, the spherical joint would be driven to generate a corresponding motion.

The orientation of the rotor is thus measured by a rotary encoder and a two-axis tilting sensor cooperating with the spherical joint. The two-axis tilting sensor is installed outside

![Figure 1. Mechanical structure of the spherical actuator.](image-url)
the rotor. This design not only simplifies the orientation measurement structure, but also decreases the size of the rotor. The inertia moment of this measurement system is much smaller compared with the conventional three-encoder system proposed in Lee et al. (2004). Furthermore, compared with the non-contact orientation measurement methods such as the Hall sensors method, optical sensors method and vision-based method, it has the advantages of no blind area, insensitivity to environment and high resolution. With this measurement method, closed-loop control method could be developed to achieve accurate positioning (Chen et al., 2012).

**Dynamic modelling**

In this section, a dynamic model of the spherical actuator is derived in detail. The mechanical structure of the rotor is shown in Figure 3. The semi-ball shaped rotor consists of eight PMs and the tilting sensor, which is appended from the spherical joint bearing. The Lagrangian $L$ is defined as

$$L(q, \dot{q}) = T(q, \dot{q}) - V(q)$$

where $T$ is the kinetic energy and $V$ is the potential energy, both represented in generalized coordinates. The generalized coordinates are $q_1 = \alpha$, $q_2 = \beta$, $q_3 = \gamma$.

The dynamic model of the rotor can be derived from the Lagrange’s equations, which are expressed in the generalized coordinates $q \in \mathbb{R}^m$ and Lagrangian $L$ is given by (Murray et al., 1994)

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i, i = 1, \ldots, m$$

where $\tau_i$ is the external force/torque on the $i$th generalized coordinate. The generalized forces/torques are

$$\tau_1 = \tau_\alpha - \tau_{ext^\alpha}, \tau_2 = \tau_\beta - \tau_{ext^\beta}, \tau_3 = \tau_\gamma - \tau_{ext^\gamma}$$

where $\tau_{ext^\alpha}$, $\tau_{ext^\beta}$ and $\tau_{ext^\gamma}$ are the extra torque in the three Euler angle axis respectively, which include the load torque and the disturbances.

The kinetic energy of the rotor in the rotor frame is given by (Wu et al., 2011):

$$\mathbf{T} = \frac{1}{2} J_{xx} \dot{\alpha}^2 + \frac{1}{2} J_{yy} \dot{\beta}^2 + \frac{1}{2} J_{zz} \dot{\gamma}^2$$

where $J_{xx}$, $J_{yy}$ and $J_{zz}$ are the principle rotational inertial moments respectively. The angular velocity vector $\omega = [\dot{\omega}_x \dot{\omega}_y \dot{\omega}_z ]^T$ can be obtained by coordinate transformation, i.e.

$$\omega = \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{bmatrix}$$

where $c$ and $s$ represent cosine and sine functions respectively. Substituting Equations (1), (3) and (4) into Equation (2) gives

$$\mathbf{M}(\mathbf{q}) \ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{\tau}_g = \mathbf{\tau}_c - \mathbf{\tau}_{ext}$$

where $\mathbf{q} = [\alpha \beta \gamma]^T$ is the orientation output expressed in Euler angles, $\mathbf{\tau}_c = [\tau_\alpha \tau_\beta \tau_\gamma]^T$ is the control torque, $\mathbf{\tau}_g = [\tau_{xg} \tau_{yg} \tau_{zg}]^T$ is the gravitational torque, $\mathbf{\tau}_{ext}$ is the external torque including the load torque and the disturbances. Here we denote the principal inertial moment as $J_{xx} = J_1, J_{yy} = J_2, J_{zz} = J_3$. The inertial matrix $\mathbf{M}(\mathbf{q})$ and the centripetal and Coriolis matrix $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$ can be obtained in the following form:

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} J_1 c^2 \beta^2 \gamma + J_2 c^2 \beta \gamma + J_3 c^2 \beta & (J_1 - J_2) c \beta \gamma & J_3 \beta \\ (J_1 - J_2) c \beta \gamma & J_1 \gamma^2 + J_2 \gamma^2 & 0 \\ J_3 \beta & 0 & J_3 \end{bmatrix}$$

$$\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

where

$$C_{11} = (J_1 c^2 \beta^2 \gamma + J_2 c^2 \beta \gamma + J_3 c^2 \beta) \beta$$

$$+ (-J_1 \gamma c^2 \beta + J_3 c^2 \beta \gamma) \gamma.$$
Parameter identification

Inverse dynamic model

In order to facilitate the identification procedure, the dynamic model (9) is reconstructed in the following form.

\[
\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{\tau}_c - \mathbf{\tau}_{cext} \tag{10}
\]

where \(\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}})\) is a (3×1) non-linear vector including the centripetal and Coriolis forces, gravity force and friction forces/torques, i.e. \(\mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})\mathbf{\tau}_v + \mathbf{\tau}_g + \mathbf{\tau}_f\).

When the dynamic model and the torque input are determined, the orientation output of the spherical actuator \(\mathbf{q} = [\alpha \ \beta \ \gamma]^T\) can be obtained by solving the differential equation (10), which can be re-written as a non-linear state-space model

\[
\dot{\mathbf{x}} = \mathbf{A}(\mathbf{x}) + \mathbf{B}\mathbf{u} \tag{11}
\]

where \(\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}\) is the system output, \(\mathbf{A}(\mathbf{x})\) and \(\mathbf{B}\) are given by

\[
\mathbf{A}(\mathbf{x}) = \begin{bmatrix} -\mathbf{C}^{-1}\mathbf{N}^j & \mathbf{0}_{3\times3} \\ 0_{3\times3} & \mathbf{M}^{-1} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} \mathbf{0}_{3\times3} \\ \mathbf{M}^{-1} \end{bmatrix} \tag{12}
\]

where \(0_{3\times3}\) is the (3×3) identity matrix.

The output equation is

\[
\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{13}
\]

where \(\mathbf{y} = \mathbf{q}\) is the system output, \(\mathbf{C} = [\mathbf{I}_{3\times3} \quad \mathbf{0}_{3\times3}]\) is the (3×3) output matrix, \(\mathbf{I}_{3\times3}\) is the (3×3) identity matrix, \(\mathbf{D} = \mathbf{0}_{3\times3}\) is the direct feedthrough matrix.

The identification of dynamic parameters is based on the inverse dynamic model of the spherical actuator, which can be obtained from the dynamic model (10), that is

\[
\mathbf{\tau}_c - \mathbf{\tau}_{cext} = \mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{N}(\mathbf{q}, \dot{\mathbf{q}}) \tag{14}
\]

By omitting the load torque and disturbances, i.e. setting \(\mathbf{\tau}_{cext} = 0\), Equation (14) can be rewritten in the following form, which is linear to the parameter vector.

\[
\mathbf{\tau}_c = \mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})^T \mathbf{x} \tag{15}
\]

where

\[
\mathbf{x} = [J_i \ \tau_{g,i} \ \tau_{f,i}]^T, \quad i = 1, 2, 3, \ j = 1, 2, 3 \tag{16}
\]

is the dynamic parameter vector having 12 parameters and \(J_i, \tau_{g,i}, \tau_{f,i}\) represents the Euler angles \((\alpha, \beta, \gamma)\) respectively, \(\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\) is the Jacobian matrix of the control torque \(\mathbf{\tau}_c\) with respect to the parameter vector \(\mathbf{x}\) shown below.

\[
\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = [\mathbf{Y}_f \ \mathbf{Y}_g \ \mathbf{Y}_\tau] \tag{17}
\]

where \(\mathbf{Y}_f, \mathbf{Y}_g\) and \(\mathbf{Y}_\tau\) are the regression matrices of the inertial moment, the gravitational and the friction torque respectively. The analytical expression of each regression matrix is given in the appendix of this paper.
1) Design a closed-loop control law to make sure that the actual torque $\tau$ differs from the control torque $\tau_c$ by an error $\rho$, i.e.

$$\tau = \tau_c + \rho = Y(q, \dot{q}, \ddot{q})\chi + \rho$$

Equation (18) is the inverse dynamic identification model of the spherical actuator.

**Identification scheme**

Usually, the dynamic parameters $\chi$ can be estimated by solving the Equation (18) when the torque input $\tau$ and the orientation output $(q, \dot{q}, \ddot{q})$ are collected while the actuator is tracking a trajectory. However, this method needs accurate measurement of both the torque and the orientation output. Inspired by Gautier et al. (2012), here we introduce a new identification method, which only needs to measure the current of the spherical actuator to estimate its dynamic parameters.

The proposed identification scheme is shown in Figure 4. This method is mainly based on OE identification and recursive LS estimation. There are two loops, namely the actual one and the simulated one. The same excitation trajectories and control law are applied to both actual and simulated systems. The estimated dynamic parameter $\chi$ can be obtained by minimizing the error between the actual and simulated current.

The procedure of the proposed method is described as follows:

1) Design a closed-loop control law to make sure that the output trajectory can well track the given trajectory.

2) Apply the designed control law in both actual and simulated systems to get the actual control current input $I_a$ and the simulated control current input $I_s$.

3) Supply the actual control current input $I_a$ to the actual spherical actuator. Then it will output the orientation $q = [\alpha \beta \gamma]^T$, which can be sampled by the orientation measurement system and feedback to the control law.

4) When the simulated control current input $I_s$ is applied to the simulated one that is the inverse dynamic model of the spherical actuator, the simulated orientation output $\hat{q}$ can be obtained by solving the differential equation (10) and then also feedback to the control law.

5) The estimated dynamic parameter $\hat{\chi}$ can be obtained by minimizing the error between the actual and simulated current input, i.e. $\hat{\chi} = \min ||I_a - I_s||^2$.

6) Update the dynamic parameter $\hat{\chi}$ in the inverse dynamic model in the next iteration.

7) Repeat the above steps until an ending condition is satisfied.

**Control law design**

The control principle of the spherical actuator is illustrated in Figure 5. The control law here contains two parts, i.e. a PD control law and an inverse torque model.

When the designed trajectory is given and the orientation output of the rotor is measured by the orientation measurement system, the control torque $\tau$ can be produced by the PD control law

$$\tau = K_p e + K_d \dot{e}$$

Equation (19) is the inverse dynamic identification model of the spherical actuator.

$$\tau = G(q)I$$

where $e = q_d - q$ is the position error, $\dot{e} = \dot{q}_d - \dot{q}$ is the velocity error, $q$ and $\dot{q}$ are the sensor-measured position and velocity variables of the rotor respectively, and $K_p$ and $K_d$ are constant gain matrices. Then the input current can be obtained by applying the control torque $\tau$ to the inverse torque model, which can be seen in our previous work (Chen et al., 2012).

The torque and inverse torque model of the spherical actuator are expressed as follows, respectively:

$$\tau = G(q)I$$

$$I = G(q)^T(G(q)G(q)^T)^{-1}\tau$$

where $G(q)$ is torque-to-current gain matrix, which is related to the orientation of the spherical actuator. It is calculated based on finite element (FE) computation and curve fitting method. Its detailed form can be seen in Zhang et al. (2011).

Combining Equations (19) and (21), the final augmented control law is

$$I = G(q)^T(G(q)G(q)^T)^{-1}(K_p e + K_d \dot{e})$$

This control law is applied to both the actual and simulated loops to calculate the control current input $I_a$ and $I_s$.

**Parameter identification algorithm**

As illustrated in the flowchart in Figure 6, the identification procedure is detailed in the following steps:
Step 1: Choose the initial value of the dynamic parameters $\hat{x}_0 = \left[ f_p \quad \tau_{pe} \quad f_{pe} \quad f_{pe}' \right]^T$.

Step 2: Sample the actual and the simulated loop at different time interval (noted $k = 1, 2, \cdots, m$) to get the actual current matrix $I_a = [I_1 \ I_2 \ \cdots \ I_m]$, the simulated current matrix $I_s = [I_1 \ I_2 \ \cdots \ I_m]$, and the corresponding orientation output $q_i = [q_1 \ q_2 \ \cdots \ q_i]$. Both current matrices are $(24 \times m)$ dimensions, while the matrix of simulated orientation output is $(3 \times m)$ dimensions.

Step 3: Calculate the current error $\Delta I_l = I_I - I_s$, and the torque error $\Delta \tau = G_i(q_i)\Delta I_l$ for $l = 1, 2, \cdots, m$, where $G_i(q_i)$ is torque-to-current gain matrix.

Step 4: Group the sampling data together to get an overdetermined equation $Y(q_i, \hat{q}_i, q_s)\Delta \hat{X} = \Delta \tau$, where

$$Y(q_i, \hat{q}_i, q_s) = \begin{bmatrix} Y_1(q_i, \hat{q}_i, q_s) \\ Y_2(q_i, \hat{q}_i, q_s) \\ \vdots \\ Y_n(q_i, \hat{q}_i, q_s) \end{bmatrix}, \quad \Delta \hat{X} = \begin{bmatrix} \Delta \hat{X}_1 \\ \Delta \hat{X}_2 \\ \vdots \\ \Delta \hat{X}_n \end{bmatrix}$$

Use the LS estimation method to calculate the incremental value of the parameters

$$\Delta \hat{X}_i = [Y(q_i, \hat{q}_i, q_s)]^{-1} \Delta \tau$$

where $[Y(q_i, \hat{q}_i, q_s)]^{-1} = (Y^T Y)^{-1} Y$ is the pseudoinverse of $Y(q_i, \hat{q}_i, q_s)$.

Step 5: Update the identified parameters $\hat{X}_i = \hat{X}_{i-1} + \Delta \hat{X}_i$ until it satisfies the ending condition

$$\max_{i = 1, \ldots, 12} \frac{|\Delta \hat{X}_i|}{\hat{X}_i} \leq e$$

where $e$ is the ending error to get fast convergence with good accuracy. If the ending condition is not satisfied, go back to Step 2. It should be kept in mind that this algorithm is effective by assuming that the simulated loop orientation output is close to the actual one at any iteration $i$, i.e.,

$$\hat{q}_i(\hat{x}_i), \hat{q}_i(\hat{x}_i), \hat{q}_i(\hat{x}_i) = (q_i, \hat{q}_i, q_s)$$

which will be discussed in detail below.

Note that the simulated loop is conducted iteratively until getting the best estimation results, while the actual currents is just sampled in one iteration, which not only facilitates the identification experiment but also introduces less measurement noise leading to more precise identification results.

**Algorithm simulation**

In this section, a simulation example is given in Matlab Simulink to demonstrate the effectiveness and robustness of the proposed identification algorithm.

The first step is to determine the excitation trajectory, which is the input of both the actual and simulated loop. According to Swevers et al. (2007) and Presse and Gautier (1993), periodic excitation enables time-domain data averaging and can improve the signal-to-noise ratio of the experimental data. Thus the excitation trajectory of this spherical actuator is chosen as

$$q_i(t) = q_{i,0} + \sum_{k=1}^{5} (a_{i,k} \sin(k\omega t) + b_{i,k} \cos(k\omega t))$$

where $a_{i,k}$ and $b_{i,k}$ are the coefficients of the sine and cosine functions, and $q_{i,0}$ is the offset angular position. The fundamental period of the above Fourier series is $2\pi/\omega_i$. Here $i = 1, 2, 3$ represents the Euler angles $\alpha$, $\beta$ and $\gamma$ respectively. The coefficients are computed by considering the constraints of the position, the velocity and the acceleration. The excitation trajectories of each axis are shown in Figure 7. Due to
the structure symmetry, the excitation trajectories in terms of \( \alpha, \beta \) Euler angles are chosen the same.

The next step is to assign the parameter errors to the initial values and conduct the simulation. Here we add the errors to the initial values of the dynamic parameters:

\[
\dot{\mathbf{x}}_0 = \begin{bmatrix} J_1 + \delta J_1 \\ \tau_{p,1} + \delta \tau_{p,1} \\ f_{c,1} + \delta f_{c,1} \\ f_{v,1} + \delta f_{v,1} \end{bmatrix}
\]  \( (27) \)

where \( \delta J_1, \delta \tau_{p,1}, \delta f_{c,1} \) and \( \delta f_{v,1} \) are the dynamic parameter errors, which are all set at 50% of the ‘actual values’, listed in Table 3.

The ‘actual values’ of the inertial moment \( J_1, J_2, J_3 \) are determined by the CAD value, while the other parameters (the gravitational and the friction parameters) are determined according to a priori knowledge and experience.

In the simulation, to imitate the actual spherical actuator system, a random disturbance torque ranging from \(-0.001 \) to \( 0.001 \) Nm is added into the inverse dynamic model of the actual loop.

The current of the stator coils is not sampled all the time during the whole experimental process, which is different from Gautier et al., (2012). Instead, we sample the current of both loops over the period, when their orientation outputs \(( \mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\) almost track the desired excitation trajectories \(( \mathbf{q}_d, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\), respectively. In this way, reasonable tracking performance is obtained without updating the coefficients of the PD control law in any iteration, in contrast to Gautier et al., (2012). Thus this would finally ensure the condition that \(( \mathbf{q}_s, \dot{\mathbf{q}}_s, \ddot{\mathbf{q}}_s) \)'s achieved. The sampling frequency is set as \( f_s = 10 \) Hz and the number of sampling is set as \( m = 30 \).

The identification errors of the inertial moment parameters during the iterative procedure of the simulation are shown in Figure 8, where (a)–(c) are the three principal inertial

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Error</th>
<th>Parameter</th>
<th>Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta J_1 )</td>
<td>0.875e-3</td>
<td>( \delta f_{c,1} )</td>
<td>2.5e-4</td>
</tr>
<tr>
<td>( \delta J_2 )</td>
<td>0.725e-3</td>
<td>( \delta f_{c,2} )</td>
<td>2.5e-4</td>
</tr>
<tr>
<td>( \delta J_3 )</td>
<td>0.725e-3</td>
<td>( \delta f_{c,3} )</td>
<td>5e-4</td>
</tr>
<tr>
<td>( \delta \tau_{p,1} )</td>
<td>5e-4</td>
<td>( \delta f_{c,1} )</td>
<td>1e-4</td>
</tr>
<tr>
<td>( \delta \tau_{p,2} )</td>
<td>5e-4</td>
<td>( \delta f_{c,2} )</td>
<td>1e-4</td>
</tr>
<tr>
<td>( \delta \tau_{p,3} )</td>
<td>5e-4</td>
<td>( \delta f_{c,3} )</td>
<td>2.5e-4</td>
</tr>
</tbody>
</table>

Figure 8. Identification errors during iterative procedure: (a) inertial moment \( J_1 \); (b) inertial moment \( J_2 \); (c) inertial moment \( J_3 \).
parameters $J_1$, $J_2$ and $J_3$, respectively. From the simulation results, we can see that after three steps of iteration, these parameters have already been close to the ‘actual value’ with the maximum relative error $e < 5\%$. After 12 steps of iteration, the maximum relative error satisfies that $e < 0.1\%$.

Figure 9(a)–(c) are the identification results of the gravitational, Coulomb friction and viscous friction parameters $\tau_g$, $f_c$ and $f_v$ during the iterative process, respectively.

The simulation results show that all these three groups of the dynamic parameters can converge to the ‘actual values’ well. The poorest identification results occur at the viscous friction parameters, where the maximum relative error is about $e = 4.5\%$ after a 12-step iteration.

The assumption (25) that the simulated loop orientation output is close to the actual one at any iteration $l$, i.e. $(q_s(\chi^l), q_s(\chi^l), q_s(\chi^l))=\langle q, q, q \rangle$ is confirmed in Figure 10, where the position errors of Euler angles $(\alpha, \beta, \gamma)$ are shown at iteration $l = 1$. The maximum relative norm error is close to 1.5% for the position, 10% for the velocity and 25% for the acceleration, which are acceptable in this algorithm. As iteration continues, these errors become smaller and smaller. All these simulation results show that the proposed identification algorithm has the ability to identify the dynamic parameters of the spherical actuator correctly even an extra disturbance torque is added.

**Experimental studies**

**Experiment set-up**

The identification experiment mainly consists of two parts, i.e. the data collection and the offline identification. The actual loop current collection is conducted on an experiment platform, while the simulated loop and offline identification
are conducted in the MATLAB Simulink environment. Figure 11 shows the diagram of the experimental platform implementation.

This experimental platform consists of a personal computer (PC) and a multi-channel current controller. The PC is mainly responsible for the implementation of control algorithm and storage of sampling data, while the current controller is in charge of driving the spherical actuator as well as sampling the current of stator coils. As illustrated in Figure 11, the multi-channel current controller contains the CPU, RS232, D/A, A/D, V/I converter and current sampling function modules. The core part of current controller is the ARM7 microprocessor LPC2138, which is mainly responsible for target scheduling while CPLD is in charge of driver function programming. The D/A chip AD5372 contains 40 16-bit bipolar DACs. A high-voltage, high-current operational amplifier OPA549 is employed to convert the analogue voltage signals from DACs to the current input. The current in each coil is detected by measuring the voltage drop of the current sampling resistor. Then the voltage signal is digitalized by the A/D chip ADS8364, which includes six 16-bit, 250 kHz ADCs with six fully differential input channels grouped into two pairs for high-speed simultaneous signal acquisition. The orientation of the spherical actuator is obtained by inputting the encoder signal into the CPLD and the digital tilting sensor signal into the ARM through the RS232 port. The research prototype and the experimental platform of the spherical actuator are shown in Figure 12.

**Experimental procedure**

The identification experiment is divided into the following two parts: 1) the actual loop current data collection while the spherical actuator is tracking the designed excitation trajectory; and 2) the simulated loop current data collection with the identified parameter iteration.
The initial step of the actual loop data collection is to tune the PD coefficients of the control law (22) in order to make the orientation output track the designed excitation trajectory well. According to our experience and several trial and errors, the PD parameters are chosen as

\[ K_p = \text{diag}(150, 100, 100), \quad K_d = \text{diag}(10, 15, 10) \quad (28) \]

The position \( q \) is available for the measurement, which can be easily obtained by the orientation measurement system designed above. The position \( q \) is then filtered by a Butterworth filter with a 20-Hz cut-off frequency, and the velocity \( \dot{q} \) and acceleration \( \ddot{q} \) are calculated with a numerical differentiation. The sampling frequency of the orientation position \( q \) is set as \( f_q = 40 \) Hz, which mainly depends on the measurement frequency of the two-axis tilting sensors (SANG1000) used in this experiment (Chen et al., 2012).

In this experiment, the current sampling frequency is set as \( f_c = 10 \) Hz in the duration \( t = [6, 9] \) s and thus the number of sampling is \( m = 30 \). Considering the simulation results and the actual factors of this research prototype, the iteration stop condition of the experiment is set as \( e \leq 5\% \).

### Experimental results

The initial values of the dynamic parameters chosen are listed in Table 4. It costs five iteration steps to obtain the experimental results of the dynamic parameters. The experimental results of the identified parameters are given in Table 5.

The identification results show that the actual dynamic parameters of the spherical actuator are different from the theoretical results, which are computed via CAD or other computer-aided design software. By our analysis, there are mainly two factors that contribute to the differences. One is the existing manufacturing and assembling error of the spherical actuator, which makes the actual device different from the CAD model that is built in the simulation environment. The
other is the experimental data collection error. As the identification experiment is conducted in the platform developed by us, experimental accuracy needs to be further improved. All these identified results will be checked and used in our future research work such as development of model-based dynamic control algorithms of the spherical actuator.

Conclusions

The main contributions of this paper include the formulation of the dynamic model and the dynamic parameter identification method of the spherical actuator, which can realize 3-DOF motion in a single joint. The dynamic model is derived from Lagrange’s equations and is augmented with the friction torque model. A new identification method of the dynamic parameters is proposed, which only needs to measure the current in the stator coils. Simulation studies conducted in MATLAB have shown that this method can effectively identify the dynamic parameters, in which even 50% errors are assigned initially. An experimental platform is designed and developed to incorporate MATLAB simulation to identify the dynamic parameters of the spherical actuator, which lay a solid foundation of the design of the model-based control strategies.

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References


Appendix

The analytical expressions of the following regression matrices \( Y_J \), \( Y_G \), \( Y_f \) are given below.

\[
Y_J = \begin{bmatrix}
Y_{J11} & Y_{J12} & Y_{J13} \\
Y_{J21} & Y_{J22} & Y_{J23} \\
Y_{J31} & Y_{J32} & Y_{J33}
\end{bmatrix},
Y_G = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix},
\]

\[
Y_f = \begin{bmatrix}
Y_{f11} & Y_{f12} & Y_{f13} & Y_{f14} & Y_{f15} & Y_{f16} \\
Y_{f21} & Y_{f22} & Y_{f23} & Y_{f24} & Y_{f25} & Y_{f26} \\
Y_{f31} & Y_{f32} & Y_{f33} & Y_{f34} & Y_{f35} & Y_{f36}
\end{bmatrix}
\]

\[
Y_{J11} = (c^2b^2\gamma)\ddot{\alpha} + (c\beta\gamma\gamma)\ddot{\beta} - 2(\gamma\gamma\gamma^2\beta)\dot{\alpha}\dot{\gamma} - (\gamma\gamma\gamma^2\beta)^2 + (c\beta^2\gamma - c\beta^2\gamma)\dot{\beta}\dot{\gamma},
\]

\[
Y_{J12} = (c^2b^2\gamma)\ddot{\beta} - 2(\gamma\gamma\gamma^2\beta)\dot{\alpha}\dot{\gamma} - (\gamma\gamma\gamma^2\beta)^2 + (c\beta^2\gamma - c\beta^2\gamma)\dot{\beta}\dot{\gamma},
\]

\[
Y_{J13} = (c^2b^2\gamma)\ddot{\gamma} + (c\beta\gamma\gamma)\ddot{\beta} - 2(\gamma\gamma\gamma^2\beta)\dot{\alpha}\dot{\gamma} - (\gamma\gamma\gamma^2\beta)^2 + (c\beta^2\gamma - c\beta^2\gamma)\dot{\beta}\dot{\gamma}.
\]
\[ Y_{12} = (c^2 \beta s^2 \gamma)\dot{\alpha} - (c\beta \gamma s^2 \gamma)\dot{\beta} - 2(c\beta s \beta s^2 \gamma)\ddot{\alpha} \dot{\beta} + (s\gamma c^2 \beta + c\gamma s c^2 \beta)\dot{\alpha} \dot{\gamma} + (s\beta c \gamma s^2 \gamma)\dot{\beta}^2 + (c\beta s^2 \gamma - c\beta c^2 \gamma)\ddot{\beta} \gamma, \]
\[ Y_{13} = s^2 \dot{\beta} \ddot{\alpha} + s\beta \dot{\gamma} + 2(c\beta s \beta)\dot{\alpha} \dot{\beta} + (s\beta + c\beta)\ddot{\beta} \dot{\gamma} \]
\[ Y_{21} = (c\beta \gamma s^2 \gamma)\ddot{\alpha} + (s^2 \gamma)\ddot{\beta} + (c\beta s \beta s^2 \gamma)\dot{\alpha}^2 + (c\beta c^2 \gamma - c\beta s^2 \gamma)\dot{\alpha} \dot{\gamma} + 2(c\gamma s \gamma)\dot{\beta} \dot{\gamma} \]
\[ Y_{22} = - (c\beta \gamma s^2 \gamma)\ddot{\alpha} + (c^2 \gamma)\ddot{\beta} + (c\beta s \beta s^2 \gamma)\dot{\alpha}^2 + (c\beta s^2 \gamma - c\beta c^2 \gamma)\dot{\alpha} \dot{\gamma} - 2(c\gamma s \gamma)\dot{\beta} \dot{\gamma} \]
\[ Y_{23} = - c\beta s \beta \alpha^2 - c\beta \dot{\alpha} \dot{\gamma} \]
\[ Y_{31} = (c\gamma s c^2 \beta)\dot{\alpha}^2 + (c\beta s^2 \gamma - c\beta c^2 \gamma)\dot{\alpha} \dot{\beta} - (c\gamma s \gamma)\dot{\beta}^2 \]
\[ Y_{32} = - (c\gamma s c^2 \beta)\dot{\alpha}^2 + (c\beta c^2 \gamma - c\beta s^2 \gamma)\dot{\alpha} \dot{\beta} - (c\gamma s \gamma)\dot{\beta}^2 \]

\[ Y_{33} = (s\beta)\ddot{\alpha} + \ddot{\gamma} \]

\[ Y_{11} = \text{sign}(\dot{\alpha}) \]
\[ Y_{12} = Y_{13} = Y_{21} = Y_{23} = Y_{31} = Y_{32} = 0 \]
\[ Y_{14} = \dot{\alpha} \]
\[ Y_{24} = \dot{\beta} \]
\[ Y_{34} = \dot{\gamma} \]

\[ Y_{15} = Y_{16} = Y_{24} = Y_{26} = Y_{34} = Y_{35} = 0 \]